Bonds and Bond Yield to Maturity

A **bond** is a long-term promissory note issued by a corporation. The associated contract, called the bond indenture, specifies that the creditor will receive regular interest payments for the term of the agreement and then receive the **face amount** of the bond. The **coupon rate** is the interest rate.

Objectives:

- Determine the market value of a bond.
- Determine the present value of a bond.
- Explore the impact of interest rates on bond values.
- Determine the net present value of a bond.

Example 1:

A ten-year $1,000 bond promises 6% per year with annual payments of $60 per year (0.06 * $1,000) for 10 years. On the maturity date, the lender will be paid the maturity value of $1,000. The total amount that the lender receives is the original $1,000 plus $600 (10 payments of $60 each). Find the market value of the bond.

The **market value** of a bond is the **present value** (discounted value) of the expected payments (interest and principal).

$$PV = \frac{60}{1.06} + \frac{60}{(1.06)^2} + \ldots \frac{60}{(1.06)^{10}} + \frac{1000}{(1.06)^{10}} = 441.61 + 558.39 = 1000$$

In the above expression, the first ten terms represent the present value of the interest payments, 441.61. The last term (1000/1.06^10 or 558.29) is the discounted value of the $1,000 lump sum payment. The **TVM Solver** will be used twice to compute the market value of the bond.

1. Press $\boxed{\text{apps}}$ and select **Finance**. Press $\boxed{\text{enter}}$ to select **TVM Solver** from the CALC menu.

   Note: The mode DECIMAL SETTING was changed to **FIX2** to round computations to two decimal places.

2. Because compounding occurs once a year, enter N = 10, I% = 6, PMT = 0, and FV = -1000.

3. Calculate the present value by placing the cursor on PV and pressing $\boxed{\text{alpha}}$ $\boxed{\text{solve}}$.

The present value of the $1,000 lump sum payment is $558.39.
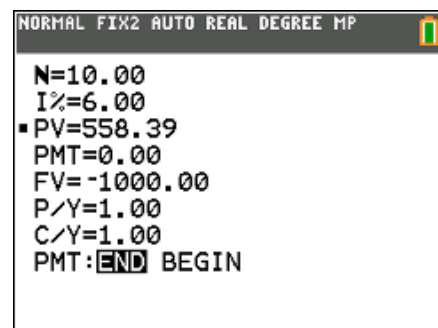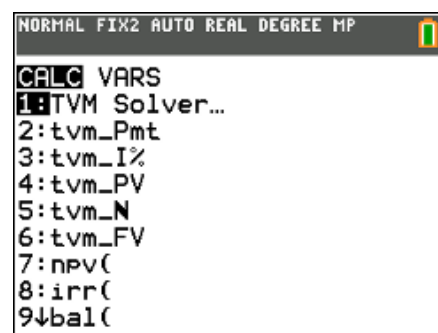
4. For the present value of the 10 payments of $60 each year, use the **TVM Solver**, and input FV = 0 and PMT = -60.

5. Calculate the present value by placing the cursor on PV and press alpha [solve].

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=10.00
 I%=6.00
•PV=441.61
 PMT=-60.00
 FV=0.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

Notice the sum of $441.61 and $558.39 equals the market value of the $1,000 bond.

This calculation could have been done in one step with the **TVM Solver**.

6. Set PMT = -60 and FV = -1000. The other values are the same as those given above.

7. Place the cursor on PV and press alpha [solve].

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=10.00
 I%=6.00
•PV=1000.00
 PMT=-60.00
 FV=-1000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```

Another method for calculating the present value is to use the **tvm_PV** command. The syntax for this function is **tvm_PV**(N, I%, PMT, FV, P/Y, C/Y).

8. Press 2nd [quit] to return to the home screen.

9. Press apps and select **Finance**. Choose **tvm_PV** from the CALC menu.

10. Enter **tvm_PV(10,6,-60,-1000,1,1)**. Press enter.

```
NORMAL FIX2 AUTO REAL DEGREE MP

tvm_PV(10,6,-60,-1000,1,1)
................................1000.00
```
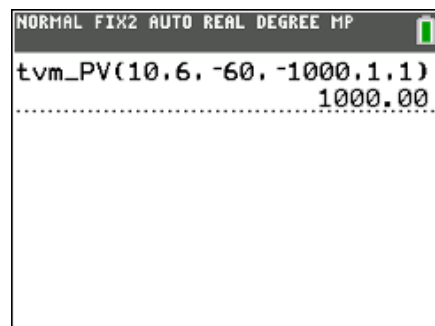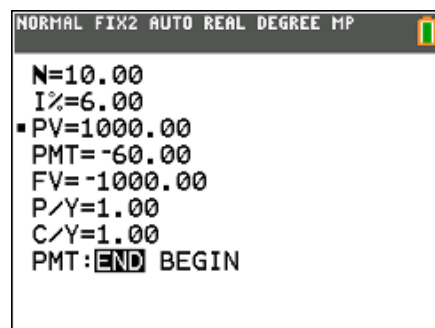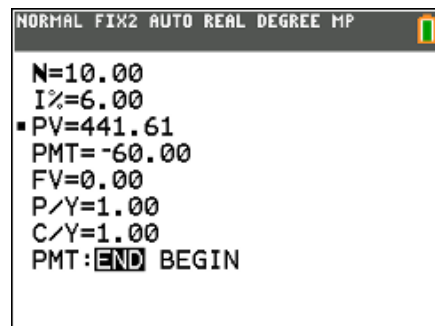
Exploration of Bond Value Versus Current Market Interest Rates

If the current market interest rate is the same as the coupon rate on bonds of similar quality, then the market value of the bond equals the face amount of the bond. If current market interest rates are higher, the market value of the bond is lower, and the bond is selling at a discount. If current market interest rates are lower, the market value of the bond is higher, and the bond is selling at a premium.
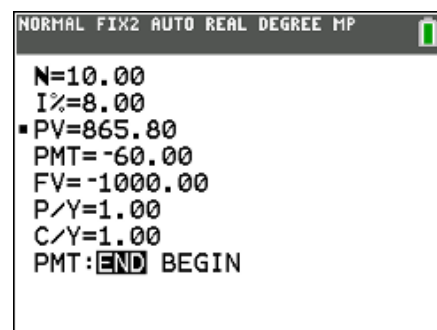
Example 2:

Find the present value of the $1,000 bond in Example 1 if the current market interest rate is 8%.

Use the **TVM Solver**.

1. Press $\boxed{\text{apps}}$, select **Finance**, and choose **TVM Solver** from the CALC menu.

2. Enter N = 10, I% = 8, PMT = -60, FV = -1000, P/Y = 1, and C/Y = 1.

3. Move the cursor to PV and press $\boxed{\text{alpha}}$ $\boxed{\text{solve}}$.

The present value of the bond is $865.80.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=10.00
 I%=8.00
▪PV=865.80
 PMT=-60.00
 FV=-1000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```
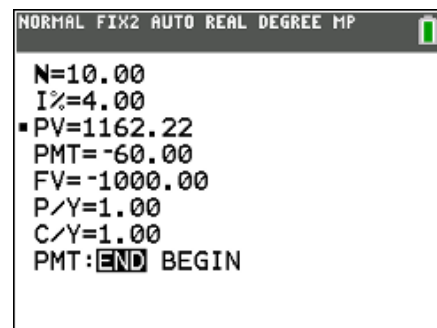
Example 3:

If the current market rate for similar bonds is 4%, what should an investor expect to pay for a $1,000, 6% 10-year bond?

1. Press $\boxed{\text{apps}}$, select **Finance**, and choose **TVM Solver** from the CALC menu.

2. Enter N = 10, I% = 4, PMT = -60, FV = -1000, P/Y = 1 and C/Y = 1.

3. Move the cursor to PV and press $\boxed{\text{alpha}}$ $\boxed{\text{solve}}$.

The investor should expect to pay $1,162.22.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=10.00
 I%=4.00
▪PV=1162.22
 PMT=-60.00
 FV=-1000.00
 P/Y=1.00
 C/Y=1.00
 PMT:END BEGIN
```
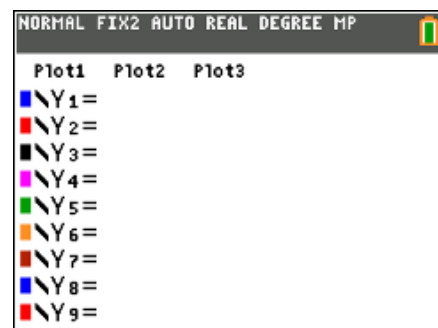
To visualize the impact of interest rate on bond values, explore the graph of the present value of the bond as a function of interest rate.

Let Y_1 be the sum of the present value of the interest payments and the present value of the $1,000 repayment of principal.
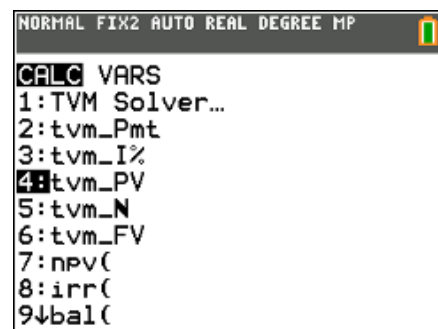
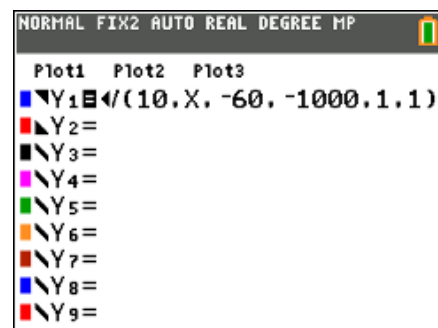Y_1 = tvm_PV(10,X,-60,-1000,1,1) where X is the interest rate.

4. To enter the function, press the $\boxed{y=}$ key. Use the $\boxed{clear}$ key to remove any functions left from another problem.
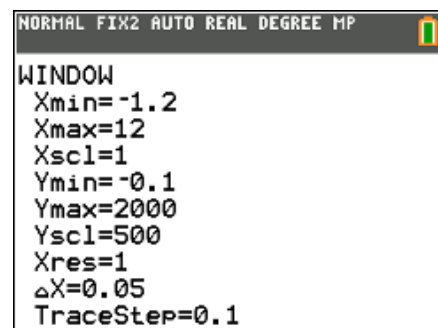
```
NORMAL FIX2 AUTO REAL DEGREE MP

 Plot1  Plot2  Plot3
■\Y₁=
■\Y₂=
■\Y₃=
■\Y₄=
■\Y₅=
■\Y₆=
■\Y₇=
■\Y₈=
■\Y₉=
```

5. Press $\boxed{apps}$, select **Finance**, and choose **tvm_PV** from the CALC menu.

```
NORMAL FIX2 AUTO REAL DEGREE MP

CALC VARS
1:TVM Solver…
2:tvm_Pmt
3:tvm_I%
4:tvm_PV
5:tvm_N
6:tvm_FV
7:npv(
8:irr(
9↓bal(
```

6. Enter **(10,X,-60,-1000,1,1)**.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 Plot1  Plot2  Plot3
■\Y₁◨◀/(10,X,-60,-1000,1,1)
■\Y₂=
■\Y₃=
■\Y₄=
■\Y₅=
■\Y₆=
■\Y₇=
■\Y₈=
■\Y₉=
```

7. Now that the function is entered, press the $\boxed{window}$ key, and enter the values shown.

```
NORMAL FIX2 AUTO REAL DEGREE MP

WINDOW
 Xmin=-1.2
 Xmax=12
 Xscl=1
 Ymin=-0.1
 Ymax=2000
 Yscl=500
 Xres=1
 ∆X=0.05
 TraceStep=0.1
```

8. Press the graph key. Use the trace key and the arrow keys to explore values. Note both X and Y values appear on the screen. Again, X is the interest rate, and Y is the present value of the bond.

    Note: Type in any X value between Xmin and Xmax, inclusive, and press enter . The X and Y values will appear on the screen.
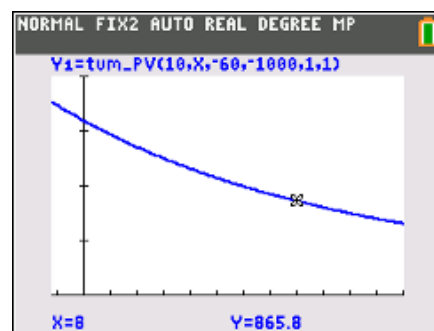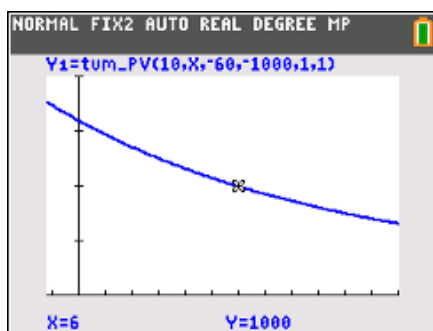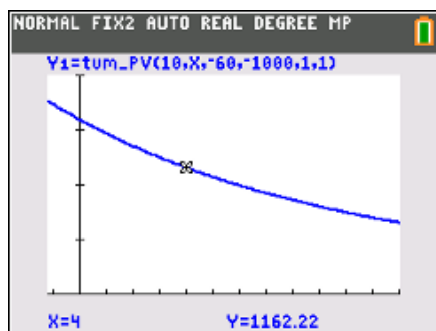
9. To see a table of interest rates and present values, press 2nd [tablset], and set TblStart to 1 and ΔTbl to 1.

10. Press 2nd [table] to display a table of the present value of the bond at various interest rates.

Example 4:

XYZ Corporation issued a 30-year $1,000 bond in 2014 with a coupon rate of 3.9%. What was it worth in 2022 if the current interest rate for similar investments in 2022 was 4.92%?

1. Press [2nd] [quit] to return to the home screen.

2. Press [apps] and select **Finance**. Choose **npv(** from the CALC menu.

   The syntax for this command is **npv**(interest rate, initial cash flow, cash flow list, frequency of cash flow entries).

   In this example, the cash flow list is $39.00 per year; the last cash flow is principal and interest, or $1,039.00. The frequency list is {21,1} because $39.00 is issued 21 times and $1,039.00 is used once.

3. Enter **npv(4.92, 0, {39, 1039}, {21,1})**. Press [enter].

The net present value in 2022 was $864.75.